Aircraft Sampling Errors for the Trade Cumulae in the Arabian Sea

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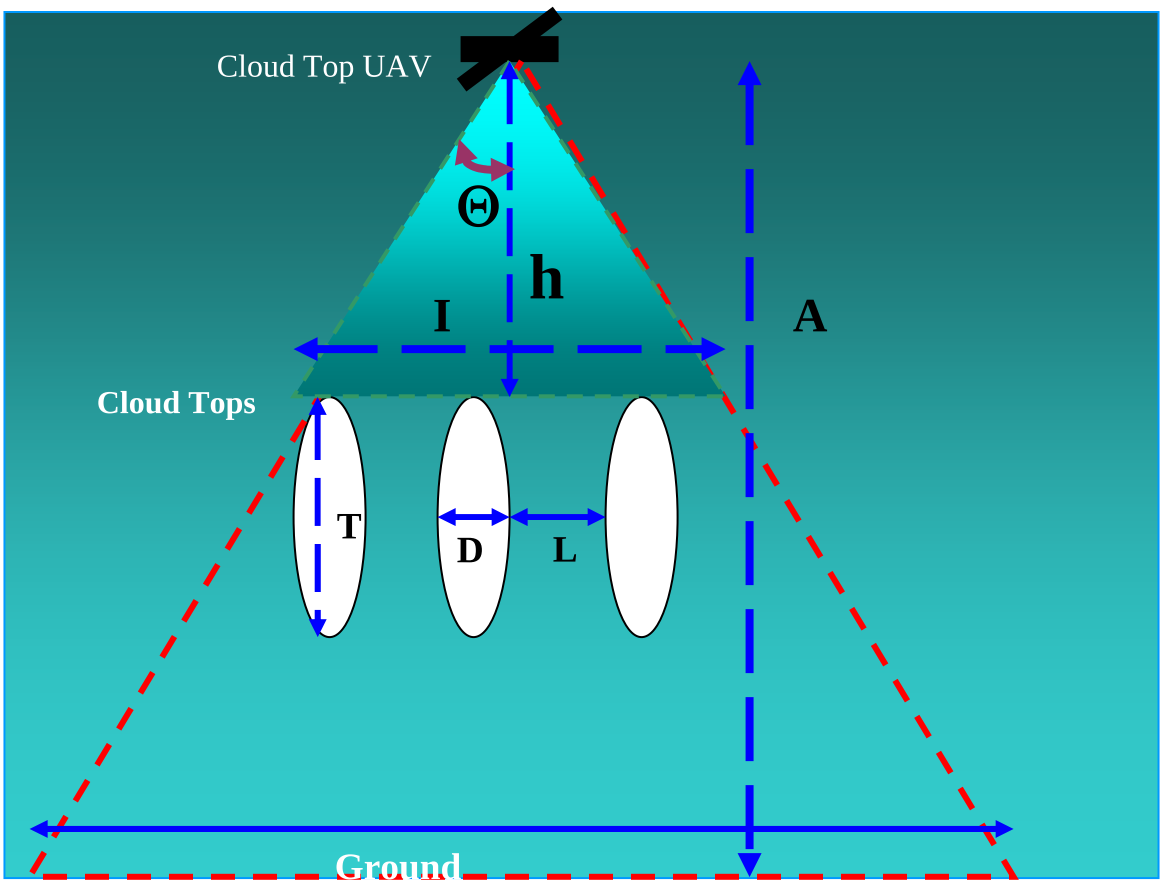
We are interested in determining the sampling error of cloudy sky albedos during the MAC campaign. An analytical derivation of the problem is given below. The equations derived can be used to maximize the collection of independent samples and to minimize the sampling error. Finally, the analysis also provides a quantification of the errors and the feasibility of observing changes in cloud forcing (i.e., average cloudy sky albedo) due to aerosol effects.



The scene above is drawn for the UAV above the cloud top, flying at **altitude A** above the ground and **height, h**, above the cloud top. The **width** of the cloud is **D**; its vertical **thickness** is T and the **distance** between each cloud is **L**.

Θ is the effective maximum zenith angle from which most of the solar radiation from cloudy sky reaches the pyranometer on the cloud top UAV. About 75% or more of the radiation incident on the pyranometer comes from a cone bounded by a zenith angle of 75 degrees or less.

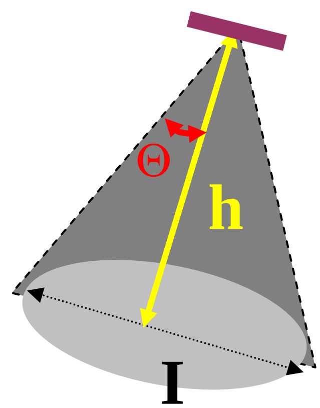
The MAC campaign is going to be sampling Trade Cus. The well mixed boundary layer thickness is about 1 km(with some diurnal change) and the trade wind inversion is about 3 km thick. The cloud base then is around 1 km and the top can vary from few hundred meters to about 3 km.

Assumption: Isotropic 3-D turbulence such that the cloud width D is about the same as cloud thickness, T; and the downdrafts and updrafts are about the same width, such that the distance between clouds is about the same as D. For the derivations given below, we need not make these assumptions, but they make the algebra easier to follow:

We have: D=L=T **(1)**

The diameter, I, of the cone of radiation seen by the UAV, from clouds located at h from the UAV altitude

$$I = 2h * Tan\Theta$$



For Tan Θ =75; $I \approx 8h$ ***(2)***

Likewise, radiation from the surface reaches the UAV from a cone with diameter

$$J \approx 8A \qquad (3)$$

The Cone area, Ca, subtended by the cone at a depth h from the UAV altitude is, using Eq. 2 for I:

$$C_a = \pi I^2/4 = \frac{\pi 64 h^2}{4} = 16\pi h^2 \qquad (4)$$

The next step is to estimate the number of cloud elements in C_a

The area of each cloud element is = $\pi D^2/4$, where we have assumed cylindrical cloud with diameter D. Assuming N clouds within the area Ca and a cloud fraction f, we obtain for the area of clouds within Ca:

$$\text{Cloud area} = f * \mathbf{C_a} = \frac{N(\pi D^2)}{4} \quad \textbf{(5)}$$

Using definition of Ca from Eq. 4, we obtain for N:

$$\mathbf{N} = 64 * f * (\frac{h}{D})^2 \quad \textbf{(6)}$$

We note that for a cloud fraction of 0.2 (taken from monthly/daily cloud fractions shown in Section 9, Mission Document) , and h=D,

$$\mathbf{N = 12 \text{ cloud elements}} \quad \textbf{(7)}$$

That is, for a cloud diameter of 1km and the UAV flying at about 1 km from the cloud, it will sample 12 cloud elements for each snapshot.

In order to estimate the number of such independent (i.e non –overlapping) cloud system that the UAV will sample in say in time interval of, δt, , we need to estimate how long it will take to travel the diameter I of the cone. If the speed of the UAV is S (km/hour), then the time, T_i, between independent sample is :

$$\mathbf{T_i = I/S = 8h/S} \quad \textbf{(8)}$$

The number of such independent samples in **, δt** is :

$$Ns = \boldsymbol{\delta t}/(8h/S) = S\frac{\delta t}{8h}$$

where time is measured in hours. For a UAV speed of 80km/hour, and h = 1km, in one hour (**δt** =1), the UAV will sample Ns= 10 cloud systems of 12 cloud elements in each system.

Thus the total number of independent cloud elements sampled in1 hour is (from Eq. 6 and 8):

$$N\,(Tot) = N * Ns = \{64 * f * (\frac{h}{D})^2\} * \frac{S * \delta t}{8h} \quad \textbf{(9)}$$

$$N\ (Tot) = 8 * f * S * \delta\mathbf{t} * \frac{h}{D^2} \qquad (10)$$

Summary

Number of cloud elements in each independent sample, captured by the pyranometer on the UAV at a distance of h from the cloud of diameter D, with a spacing D between each cloud, with the UAV traveling at a speed of S (km/s) is:

$$\mathbf{N} = 64 * f * \left(\frac{h}{D}\right)^2 \qquad \mathbf{(11)}$$

The number of such systems with N cloud sampled by the UAV in δt hours is

$$Ns = \frac{S * \delta t}{8h} \qquad \mathbf{(12)}$$

The total number of cloud elements sampled in a time interval of δτ is:

$$N\ (Tot) = N\ (Tot) = 8 * f * S * \delta\mathbf{t} * \frac{h}{D^2} \qquad \mathbf{(13)}$$

For a UAV traveling at a speed of 80 km/hr, and with h=D=1 km and a cloud fraction of 0.2 (typical numbers for MAC) the total number of clouds sampled in **δt** =1 hour is:

$$\mathbf{N\ (Tot) \approx 10^2}$$

Sampling Errors in Cloudy Sky Albedo:

Now we are ready to estimate the errors in determining the cloud forcing, i.e., the cloudy sky abedo for a region with cloud fraction f:

We let clear sky albedo, a_{clr} is 0.15; the albedo of an overcast sky, a_{ov}, the cloudy sky albedo is given by:

$$a_{cdy} = \{a_{clr}(1-f)+fa_{ov}\} \qquad (14)$$

where f is cloud fraction. The cloud forcing, C_f , is given by:

$$C_f = S(a_{cdy} - a_{clr})$$

where S is the diurnal average insolation at TOA.

The albedo of the sky overcast with one trade cu cloud can vary from about 0.15 (equal to clear sky) to about 0.6, due to variations in cloud water content, cloud depth, CCN and vertical velocity. The question is: How accurate can we determine the average cloudy sky albedo, $\langle a_{cdy} \rangle$:

$$\langle a_{cdy} \rangle = \{\langle a_{clr} \rangle (1-f) + \langle f a_{ov} \rangle\} \qquad (15)$$

where

$$\langle f a_{ov} \rangle = f\underline{a} \pm f \frac{\sigma(a_{ov})}{N(Tot)^{0.5}} \qquad (16)$$

Where σ is the standard deviation of the albedo of the trade cu albedo which (as mentioned above) can vary from 0.15 to 0.6. To keep the algebra simple, we are ignoring the term involving the product of variations in f and albedo. We will comment on that later. Upon combining Eq (13), (14), and (16):

$$\langle a_{cldy} \rangle = \{a_{clr}(1-f) + f\,\underline{a}\} \pm \frac{f(\sigma\{a_{ov}\})D}{\sqrt{8 * f * S * \delta t * h}} \qquad (17)$$

For f=0.2, h=D=1 km, S=80 km/Hr, a_{clr} =0/15, $\underline{a}$ = 0.4, we get the following for a 1 hour sampling:

$$\langle a_{cldy} \rangle = 0.2 \pm 0.005$$

If we allow for the variations in both cloud fraction and cloud albedo, the error doubles, and we get:

$$\langle a_{cldy} \rangle = 0.2 \pm 0.01$$

We are interested in detecting changes in cloud forcing (i.e,cloudy sky albedo) due to changes in aerosol optical depth (AOD). As shown in Section 9 of Mission planning document, the AOD varies from about 0.1 to 0.6 and based on the results given in Ramanathan et al, the changes in cloud forcing is about 15 Wm^{-2} or about +0.03 in cloudy sky albedo, which is a factor of 3 larger than the error estimated for a one hour sampling. Of course we can minimize the error be taking a four hour sampling, but we will loose the number of samples used to detect the change in cloud forcing with aerosol concentration, by a factor of 4.

If we sample for 4 hours each day, and we conduct the mission for about 20 days (during the 6 weeks), we will have a total of 80 samples with an error of 0.01 in cloud sky albedo, to understand the relative influence of anthropogenic effects and large scale dynamics (cloud water content, super saturation, cloud fraction) on the cloud forcing or cloudy sky albedo.

A similar analysis applies to cloud forcing at the surface (measured by ABC_MCOH) or at the cloud base by the UAV. By comparing the cloud forcing at the Cloud top UAV altitude with that at the surface, we will be able to estimate the difference between the two levels, which is the enhancement in cloudy sky absotprion due to clouds. By comparing this enhancement as a function if black carbon measured at the surface and in-situ, we will be able to provide some glimpse into the role of BC in cloudy sky absorption within experimental and sampling errors.